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Research on production strategy optimization based on unconstrained programming models

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Abstract. In the field of discrete manufacturing, particularly during electronic product assembly, quality fluctuations and cost control represent core challenges for enterprises. This paper addresses multi-stage production decision-making by constructing an integrated decision model that combines hypothesis testing, unconstrained optimization, and genetic algorithms. This model provides end-to-end support spanning sampling inspection, production process optimization, and robust decision-making under uncertainty. The model aims to maximize average profit by introducing 0-1 decision variables to characterize inspection and processing choices at each stage. It employs hypothesis testing to design minimum sample size sampling schemes, utilizes unconstrained programming for single-process and multi-process production decisions, and leverages genetic algorithms to solve large-scale combinatorial optimization problems. Under uncertainty, it characterizes defect rate fluctuations through confidence intervals, establishes robust decision models, and conducts sensitivity analysis.Numerical experiments demonstrate that the proposed model delivers effective and robust decision solutions across diverse scenarios, providing enterprises with systematic theoretical methods and practical tools to enhance quality control and economic efficiency.

Keywords: production strategy optimization, unconstrained programming, genetic algorithm, multi-stage decision-making, quality control

1. Introduction

As competition in manufacturing intensifies and consumer quality demands rise, enterprises must implement refined quality management throughout the entire production process. Particularly in discrete manufacturing processes like electronic product assembly, multiple interdependent factors—including raw material quality, assembly techniques, and post-sales handling—make traditional experience-based decision-making ill-suited for dynamically changing production environments. How to maximize profits while ensuring quality through scientific modeling and optimization methods has become a critical issue for enhancing corporate core competitiveness. This study expands upon Problem B proposed by the National College Students Mathematical Modeling Competition Organizing Committee [1], focusing on multi-stage decision optimization in electronic product manufacturing.

Quality control and decision optimization in production processes represent a significant research focus within industrial engineering and operations management. Multi-stage production quality control involves coordinated decision-making across multiple stages, including raw material inspection, in-process inspection, final inspection, and nonconforming product handling. In early research, Porteus [2] modeled quality inspection as a Markov decision process, establishing the analytical foundation based on dynamic programming. As manufacturing systems grew more complex, scholars began integrating decisions such as inspection, maintenance, scrapping, and rework while considering quality transfer effects between processes. For instance, Li et al. [3] developed a joint inspection and maintenance strategy model for serial production lines, employing stochastic optimization to minimize long-term operational costs. In recent years, domestic scholars have also explored this topic from a system optimization perspective. For instance, Li et al. [4] conducted systematic modeling and simulation analysis of inspection and maintenance decisions in multi-process production in their study Research on Multi-stage Quality Control and Cost Optimization in Manufacturing Systems Engineering. Nevertheless, most studies still assume inspections at each stage are mandatory operations, with relatively limited research on elastic optimization for the binary decision of whether to inspect. Particularly when considering multiple economic factors such as inspection costs, disassembly recycling value, and market replacement losses, a universal decision optimization framework has yet to be established. Consequently, existing research still suffers from systemic modeling deficiencies, insufficient integration of economic objectives and soft costs, and poor alignment between statistical inference and production decision-making.

This paper introduces 0-1 decision variables to characterize the detect/disassemble choices at each stage. Aiming to maximize average profit, it comprehensively employs hypothesis testing, unconstrained programming, and genetic algorithms to establish a complete modeling and optimization framework spanning from sampling inspection to multi-stage production decisions, and from deterministic optimization to robust analysis. A genetic algorithm-based solution strategy is proposed to address the problem of large-scale discrete decision combination explosion.

2. Model development

2.1. Establishing an unconstrained planning model for multi-process operations

The problem aims to investigate decision-making schemes when the number of processes is m and the number of parts is n. To this end, this paper first focuses on a specific example—two processes and eight parts—structuring, modularizing, and hierarchizing the problem. Through a two-tier assembly structure of semi-finished product → finished product, it achieves modeling expansion from single-process to multi-process scenarios. Models are subsequently established for these two critical stages: semi-finished product manufacturing and semi-finished product assembly. These models are integrated in the final phase to achieve optimization of the entire production process.

(1) Semi-finished Product Production Model

Although the components required for semi-finished products 1, 2, and 3 differ, the overall modeling process is similar. Therefore, we will focus on the modeling process for semi-finished product 1 as a representative example for detailed discussion.

First is the calculation of part costs: D_{11}.

$$D_{11} = (\alpha_1 + \alpha_2 + \alpha_3) * M_{11} \tag{1}$$

Inspection Fee: D_{12}.

$$\begin{aligned} D_{12} = [&\beta_1 * x_1 * (1 - x_2)(1 - x_3) + \ldots + x_1 * x_2 * (1 - x_3)(\beta_1 + \beta_2) \\ &+ \ldots + (\beta_1 + \beta_2 + \beta_3) * x_1 * x_2 * x_3] * M_{11} + M_{12} * y_{11} * \delta_1 \end{aligned} \tag{2}$$

Number of semi-finished product 1 shipped, number of qualified items, number of unqualified items: M_{12}, M_{13}, m_{11}.

$$\begin{cases} M_{12} = [(1-P) * x_1(1-x_2)(1-x_3) + \ldots + (1-P)^2 x_1 x_2 (1-x_3) \\ + \ldots + (1-P)^3 x_3 x_2 x_1 + (1-x_1)(1-x_2)(1-x_3)] \bullet M_{11} \\ M_{13} = (1-P)^4 M_{11} \\ m_{11} = (M_{12} - M_{13}) * y_{12} \end{cases} \tag{3}$$

Assembly cost and disassembly cost for Semi-finished Product 1: D_{14}, D_{13}.

$$\begin{cases} D_{14} = [M_{13} * y_{12} + M_{12} * (1-y_{12})] * \mu \\ D_{13} = m_{11} \cdot (\gamma_i - x_1\alpha_1 - x_2\alpha_2 - x_3\alpha_3) \end{cases} \tag{4}$$

Based on the above calculations, the various costs for semi-finished product 1 can be determined.

(2) Production model for finished goods

The process of assembling semi-finished products into finished goods is treated as a single operation, similar to the previous model where three components were assembled into semi-finished product 1. By refining the earlier model, we can construct a model for assembling semi-finished products into finished goods.

(I) Identify decision variables

Whether parts 1 to 8 are inspected (x_1), whether semi-finished products are inspected (x_2), whether non-conforming semi-finished products are disassembled (x_3), whether returned non-conforming products are disassembled (x_4), whether finished products are disassembled (x_5), and whether finished products are inspected (x_6) are still used as binary decision variables.

(II) Setting the Objective Function

This problem transforms the decision-making scheme into solving for average profit, with average profit serving as the objective function. This approach avoids direct calculation of specific part quantities, effectively circumventing the complexity of counting parts. It renders the problem more solvable and allows direct evaluation and optimization of overall profit based on finished product quantities. Average profit varies under different scenarios. This paper ultimately seeks the maximum average profit to derive a decision-making solution that better aligns with corporate development. The formula for maximum average profit is as follows:

$$\max \omega = \frac{(r \cdot M_{43} - \sum_1^6 D_{4j})}{M_{41}} \tag{5}$$

Here, $r$represents the selling price of the final product, and D_{4j} denotes the purchase cost.

Parts Purchase Cost: D_{41}.

$$D_{41} = D_{11} + D_{21} + D_{31} \tag{6}$$

Total Testing Fee: D_{42}.

$$D_{42} = D_{12} + D_{22} + D_{32} + M_{42} * y_{41} * \delta_4 \tag{7}$$

Total Disassembly Cost: D_{43}.

$$D_{43} = D_{13} + D_{23} + D_{33} + m_{41} * (\gamma_i - \sum_1^8 x_i\alpha_i) \tag{8}$$

The above formula includes the relevant counts of finished products, encompassing the number of products sold, the number of nonconforming products, and the number of products returned to the factory. Next, we calculate the relevant counts of finished products, specifically: the total number of finished products (M_{42}), the number of conforming finished products (M_{43}), the number of nonconforming finished products detected

(m_{11}), the number of finished products shipped from the factory (M_{44}), and the number of nonconforming finished products returned to the factory (m_{42}):

$$\begin{cases} M_{42} = min(M_{12}, M_{22}, M_{32}) \\ M_{43} = M_{42}(1-P) \\ M_{44} = M_{43} * y_{42} + M_{42} * (1-y_{42}) \\ m_{11} = (M_{42} - M_{43}) * y_{42} \\ m_{42} = M_{44} - M_{43} \end{cases} \tag{9}$$

In addition to the aforementioned costs, assembly expenses incurred when semi-finished products are assembled into finished goods (D_{44}) are also considered costs. Replacement losses resulting from exchanging non-conforming finished goods after shipment are also classified as costs. Furthermore, the costs associated with dismantling non-conforming finished goods upon their return to the factory remain part of the cost structure. The specific calculation is as follows:

$$\begin{cases} D_{44} = [M_{43} * y_{42} + M_{42} * (1-y_{42})] * \mu \\ D_{45} = m_{42} * \lambda_i \\ D_{46} = m_{42} * (\lambda_i + \gamma_i) \end{cases} \tag{10}$$

Finally, by integrating the calculations from the semi-finished product formula and the finished product formula, we can determine the optimal average profit for the production process involving 8 components and 2 manufacturing steps, thereby deriving the corresponding production decision plan.

(3) Model Extension

By establishing the model for the two processes and eight parts described above, it can be extended to a model for m processes and n parts.

Based on the probabilistic decision analysis method [5], the following diagram illustrates the process of detecting and disassembling multiple decision variables, as shown in Figure 1.

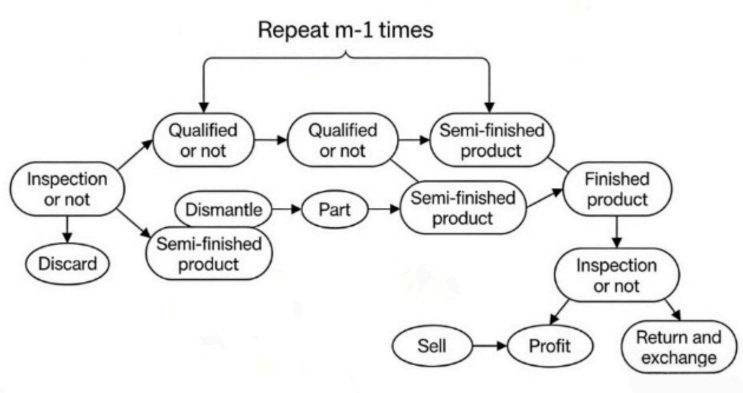

Figure 1. Multi-process planning flowchart

After analyzing the above flowchart, we established the corresponding unconstrained optimization model. First is the cost formula:

$$\begin{cases} D_{examine,i} = \sum_{j=1}^{n} \beta_j \cdot x_{i,j} \cdot M_i \\ D_{assemble,i} = \sum_{j=1}^{n} \mu_j \cdot x_{i,j} \cdot (1-P_j) \cdot M_i \\ D_{disassemble,i} = \sum_{j=1}^{n} \gamma_j \cdot x_{i,j} \cdot P_j \cdot M_i \\ D_i = D_{examine,i} + D_{assemble,i} + D_{disassemble,i} \end{cases} \tag{11}$$

Next is the average return formula:

$$\omega_{max} = \frac{r \cdot M - \sum_{i=1}^{m} D_i}{M} \tag{12}$$

Similarly to the example model, the costs associated with assembling components into semi-finished products are then calculated, ultimately yielding the average profit.

By continuing to utilize genetic algorithms for optimization, calculating all possible combinations $x_{i,j}$ and identifying the optimal combination ${x^*}_{i,j}$, the final optimal average return is: $x_{i,j}, {x^*}_{i,j}$.

$$\omega_{max} = \frac{r \cdot M - \sum_{i=1}^{m} D_i({x^*}_{i,j})}{M} \tag{13}$$

2.2. Data-driven robust decision models

Building upon the previous question, the introduction of parameter uncertainty extends the defect rate from a deterministic value to a confidence interval estimate, thereby constructing a data-driven robust decision model. The research employs the following methodological framework:

(1) Interval estimation method for constructing uncertain parameters

Based on the sampling inspection results from Problem 1, the uncertainty in the defect rate is expressed using a confidence interval:

$$P_i \in \left[\widehat{p_i} - Z_{\frac{a}{2}} \sqrt{\frac{\widehat{p_i}\left(1 - \widehat{p_i}\right)}{n}}, \widehat{p_i} + Z_{\frac{a}{2}} \sqrt{\frac{\widehat{p_i}\left(1 - \widehat{p_i}\right)}{n}} \right] \tag{14}$$

$Z_{a/2}$, the critical value for the standard normal distribution corresponding to a 95% confidence level, is obtained from the table, where $Z_{a/2} = 1.96$.

(2) Reconstruction of Optimization Models Under Interval Parameters

Replace the deterministic defect rate in the original model with an interval parameter, preserving the model structure while altering the parameters nature to form a parameter-uncertain programming model. This model simultaneously accounts for both the best-case and worst-case scenarios of the parameter. The final model is as follows:

Objective Function:

$$\max \omega(\Delta P) = \frac{r \cdot M_{43}(\Delta P) - \sum_{1}^{6} D_{4j}(x,y,z,\Delta P)}{M_{41}} * \left(1 + \sum_{1}^{\infty} J(\Delta P)^n\right) \tag{15}$$

Key Intermediate Variables (Interval Form)

Number of qualified finished products:

$$M_{43}(\Delta P) = M_{42} \cdot (1 - \Delta P) \tag{16}$$

Return rate:

$$J\left(\Delta P\right) = \frac{n_2(\Delta P)}{N_4(\Delta P)} = 1 - \frac{N_3(\Delta P)}{N_4(\Delta P)} \tag{17}$$

Maintain the 0-1 decision variables consistent with the original model, and combine them with the aforementioned model to solve the problem.

3. Model solution and sensitivity analysis

3.1. Genetic algorithms

For the multi-process production decision-making model established in this paper, which involves numerous binary decision variables and complex cost-benefit coupling relationships, direct solution attempts face combinatorial explosion issues. To address this, a genetic algorithm solution framework tailored for production optimization problems is designed and implemented, with the following specific steps: Step 1: Each candidate decision scheme is encoded as a fixed-length binary string. For the 2-process, 8-part example, a 12-bit encoding is used to sequentially represent: whether parts 1–8 are inspected, whether semi-finished products are inspected, whether non-conforming semi-finished products are disassembled, whether finished products are inspected, and whether finished products are disassembled. An initial population of 120 individuals is randomly generated, with each individual representing a complete combination of production, inspection, and disassembly decisions. Step 2: The fitness function is directly defined as the models average profit function, serving as the objective value. For each individual in the population, the corresponding inspection, disassembly, and assembly decisions are parsed from its encoding. These decisions are substituted into the established unconstrained programming model to calculate the material costs, inspection fees, disassembly recycling revenue, and final sales profit. The resulting average profit for that solution is then used as the individuals fitness value. Step 3: Selection: Employ a roulette wheel selection mechanism to choose parent individuals based on fitness value ratios, ensuring the retention of superior genes. Crossover: Perform multi-point crossover operations (with a crossover rate of 0.7) on the selected parent individuals. By exchanging segments of genetic material, new individuals are generated to explore novel decision combinations. Mutation: Randomly flip certain gene bits with a 0.01 probability to maintain population diversity and prevent premature convergence. Step 4: Repeatedly perform fitness evaluation, selection, crossover, and mutation operations for 35 generations. As evolutionary generations increase, the populations average fitness gradually improves, eventually stabilizing near the optimal value.

The entire solution process is illustrated in Figure 2 below:

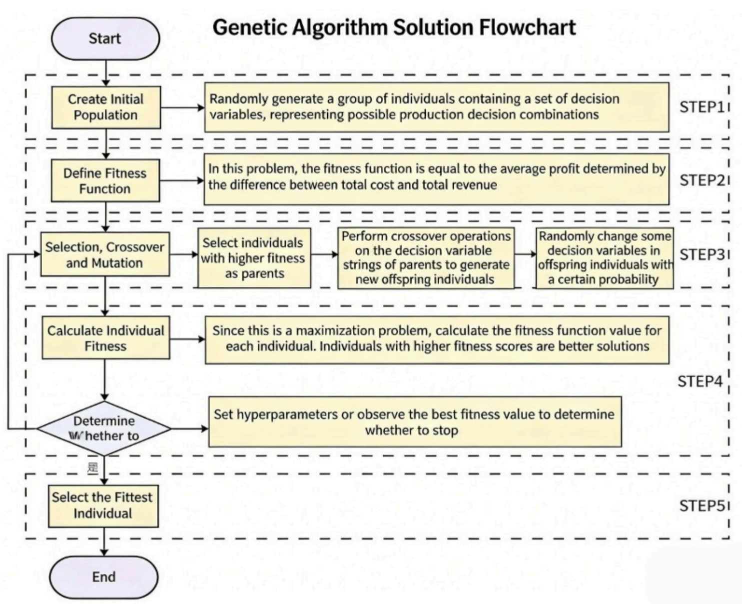

Figure 2. Flowchart of genetic algorithm solution process

3.2. Genetic algorithms for solving optimal solutions

After multiple iterations of optimization, with a population size of 120,35 iterations, a mutation rate of 0.01, a crossover rate of 0.7, and 12 variables, the genetic algorithm converged to the optimal solution. The optimal solution is [1,0,0,0,0,0,0,0,0,0,0,0], corresponding to an optimal average profit of 111 yuan.This result demonstrates that under the current parameter configuration, the genetic algorithm can effectively maximize corporate production profits while simultaneously satisfying inspection and assembly decisions for parts, semi-finished products, and finished goods.

Next, improve the code in Section 2.1 by replacing P_i with ΔP_i and adding a plotting function to obtain the graph of average profit versus defect rate. This represents the sensitivity analysis of average profit to defect rate, as shown in Figure 3.

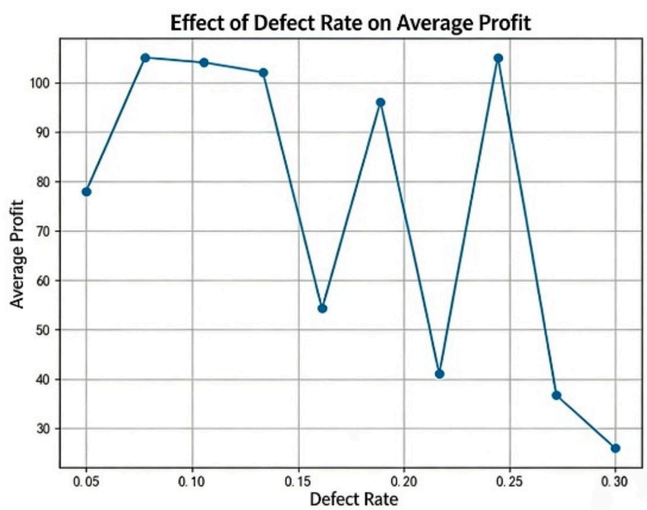

Figure 3. Average profit vs. defect rate curve

Observing the chart above, the trend of average profit as the defect rate increases is no longer a smooth curve. Possible reasons for this fluctuation include the following: Initial Stage: When the defect rate is low, its impact on average profit may be less noticeable because the quantity of nonconforming products is small and the increase in costs remains manageable. Intermediate stage: As the defect rate rises, average profit begins to decline significantly. This occurs because the rate of cost increase starts to exceed the rate of revenue decrease. High defect rate stage: When the defect rate reaches a very high level, average profit may plummet sharply into negative territory. At this stage, the company may face losses due to high costs and low sales revenue.

4. Conclusion

This paper constructs an unconstrained programming model for multi-process production systems and systematically elaborates on the process and results of solving this model using genetic algorithms. Research indicates that genetic algorithms possess the following distinct advantages in addressing such production optimization problems involving large-scale 0-1 variables:

First, in multi-process, multi-part production systems, the number of decision variables grows exponentially with scale, making traditional optimization methods prone to local optima. Genetic algorithms, through population initialization and crossover/mutation mechanisms, perform parallel, adaptive searches across vast solution spaces, effectively approximating global optima. The successful identification of the profit-maximizing inspection and disassembly decision combination in this papers case study serves as clear evidence. Second, the binary encoding of genetic algorithms inherently aligns with binary decision logic such as detect or do not detect and disassemble or do not disassemble; making the encoding intuitive and easy to interpret. Various cost and benefit constraints in this model can be naturally integrated through the fitness function without complex conversions, enhancing the algorithms practicality and scalability. Third, when production parameters (such as defect rates, market prices, or disassembly costs) change, only the corresponding components within the fitness function need adjustment. The genetic algorithm can then continue evolving from the existing population, rapidly generating optimized strategies for the new environment. This feature enables the model presented in this paper to handle not only deterministic optimization problems but also robust decision-making requirements under uncertain conditions. Fourth, the modeling-coding-evolutionary optimization framework adopted in this paper is generalizable and can be extended to other multi-stage, multi-option discrete decision-making scenarios, such as supply chain scheduling, maintenance strategy optimization, and resource allocation problems. It provides methodological references for combinatorial optimization in related fields.

In summary, the integrated framework of unconstrained programming + genetic algorithm,constructed in this paper not only provides scientific decision-making support for specific production scenarios but also offers a reusable methodological approach for similar multi-stage, multi-option discrete optimization problems. Future research may further integrate technologies such as deep learning and multi-objective optimization to expand the models application prospects in more complex manufacturing systems.

References

[1] National College Students Mathematical Modeling Competition Organizing Committee. (2024, September 5). *2024 Higher Education Press Cup National College Students Mathematical Modeling Competition Problems.* https: //www.mcm.edu.cn/html_cn/node/a0c1fb5c31d43551f08cd8ad16870444.html

[2] Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research, 34*(1), 137–144.

[3] Li, X., Wang, Z., & Chen, L. (2020). Joint inspection and maintenance policy for serial production lines with quality deterioration. *European Journal of Operational Research, 284*(1), 220–234.

[4] Li, K., Zhang, M., & Wang, L. (2023). Research on multi-stage quality control and cost optimization in manufacturing systems engineering. *Systems Engineering Theory and Practice, 43*(5), 128–137.

[5] Ding, Z. J., & Zheng, Z. Y. (2008). *Probability theory and mathematical statistics: Applications in decision making* (2nd ed.). Science Press.